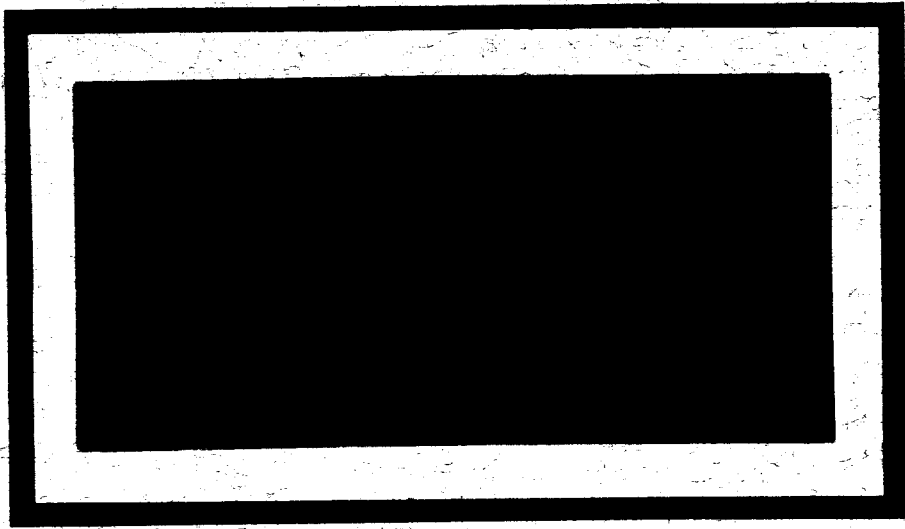6

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ELECTRON DENSITY IN THE IONOSPHERE

Quarterly Engineering Progress
Report No. 4)

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1. Introduction

The two payloads and auxilliary equipment were taken to Wallops Island, Virginia, on 25 May 1960, in expectation of firing the first on 1 June 1960. Preparation of the payload was almost complete when, on 27 May 1960, we were informed that no further Nike-Asp firings would be allowed until the cause of certain previous rocket failures (not on this project) had been eliminated. The equipment was therefore removed from Wallops Island.

The remainder of this report consists of an original contribution to the theory of the rocket-borne Langmuir probe; the current-voltage characteristic has been derived as a function of the area ratio of the two electrodes. The limiting case of a symmetrical bipolar probe (area ratio unity) has been discussed by Johnson and Malter. When the area ratio is very large the current-voltage characteristic becomes essentially that of a single probe as discussed by Langmuir. The intermediate cases particularly when the area ratio is of the same order as the ratio of random current density of electrons and positive ions (about 200 in the ionosphere) is of great importance in designing and interpreting rocket experiments.

2. The Asymmetrical Bipolar Probe

We consider an asymmetrical bipolar probe consisting of two electrodes of unequal surface area between which a potential may be applied and the resulting current measured. It is assumed that (1) the electron velocity distribution is Maxwellian and can be represented by the electron temperature

T_e and (2) the sheath thickness is small compared with the radius of curvature of the electrodes.

Initially, with both electrodes at the same potential, the system assumes a negative potential with respect to the plasma due to electron and positive ion bombardment. This floating potential is given by

$$V_f = -(kT_e/e) \log_e (j_e/j_p) \tag{1}$$

where k is Boltzmann's constant

T_e the equivalent electron temperature

e the electronic charge

j_e the electron random current density

j_p the positive ion random current density.

A potential V is applied between the electrodes, V being taken to be positive when the smaller (area A_2) is positive with respect to the larger (area A_1). The potentials of the electrodes assume new values V_1 and V_2 , where

$$V = V_2 - V_1 \tag{2}$$

The electron currents to the electrodes are, for retarding potentials,

$$i_{e1} = A_1 j_e \exp \left[eV_1/kT_e \right] \tag{3}$$

$$i_{e2} = A_2 j_e \exp \left[eV_2/kT_e \right] \tag{4}$$

where the subscripts 1 and 2 refer to the larger and smaller electrode respectively.

It is convenient at this point to introduce two parameters: the area ratio:

$$\sigma = A_1/A_2 \tag{5}$$

and the voltage ratio:

$$\eta = eV/kT_e \tag{6}$$

with appropriate subscripts for V being applied to η. Thus η is the potential V expressed as a multiple of the electron energy in voltage units.

Now the positive ion currents to the two electrodes are independent of V_1 and V_2 and have values

$$i_{p1} = A_1 j_p \tag{7}$$

and

$$i_{p2} = A_2 j_p \tag{8}$$

The total electron current must be numerically equal to the total positive ion current, that is

$$i_{e1} + i_{e2} = i_{p1} + i_{p2} \tag{9}$$

From (3) and (4):

$$i_{e1}/i_{e2} = (A_1/A_2) \exp \left[e \left(V_1 - V_2 \right) / kT_e \right] \tag{10}$$

that is

$$i_{e1}/i_{e2} = \sigma \exp \left[-\eta \right] \tag{11}$$

and from (7) and (8):

$$i_{p1}/i_{p2} = \sigma \tag{12}$$

Finally the probe current i is given by

$$i = i_{p1} - i_{e1} \tag{13}$$

Therefore, using (9), (11), (12), and (13), we find:

$$i/i_{pl} = \left\{ \frac{\exp(\eta) - 1}{\exp(\eta) + \sigma} \right\} \tag{14}$$

This is the mathematical formulation of the current-voltage characteristic of an asymmetrical bipolar probe having an area ratio σ ($\geqslant 1$). It is plotted in Figure 1 for values of σ of 1, 10, 100, and 1000. The curve for the symmetrical bipolar probe ($\sigma = 1$) is point symmetric about the origin (only the positive half is shown). For values of σ greater than 100, the curves become point symmetric about the point

$$i/i_{pl} = 0.5 \tag{15}$$

and

$$\eta = \log_e(2 + \sigma) \tag{16}$$

We must now introduce the restriction that the above analysis is valid only for retarding potentials on the electrodes. For accelerating potentials, the electron current is limited to the random current density. Now the potentials of the two electrodes η_1 and η_2 are given by

$$\exp(-\eta_1) = (j_e/j_p) \frac{\sigma + \exp(\eta)}{\sigma + 1} \tag{17}$$

and

$$\exp(-\eta_2) = (j_e/j_p) \frac{\sigma \exp(-\eta) + 1}{\sigma + 1} \tag{18}$$

Saturation occurs when the smaller electrode reaches space potential ($\eta_2 = 0$) at a value of η given by

$$\exp(\eta) = \frac{\sigma (j_e/j_p)}{\sigma + 1 - (j_e/j_p)} \tag{19}$$

and a value of i given by

$$i/i_{p1} = (j_e/j_p - 1) / \sigma \qquad (20)$$

If $j_e/j_p > (\sigma + 1)$, no electron current saturation occurs.

The point at which electron current saturation occurs is thus determined by the area ratio σ and the random current density ratio j_e/j_p. The short horizontal lines in Figure 1 indicate electron current saturation. In the ionosphere the value of j_e/j_p is computed to be about 200.

The floating potential is given by

$$\exp(\eta_f) = j_e/j_p \qquad (21)$$

and therefore, as might be guessed, for very large σ $(\gg j_e/j_p)$ the smaller electrode saturates when η is equal to the floating potential. In other words, for sufficiently large σ the smaller electrode is essentially a single probe and the larger electrode maintains a constant potential equal to the floating potential. We may determine how large σ must be for such a simplification to be made; it evidently must be several times the value of j_e/j_p. In Figure 2 the curves of Figure 1 are replotted on semi-log paper. On this plot the ordinate is

$$\Delta\, i/i_{p1} = (i + i_{p2}) / i_{p1} \qquad (22)$$

The single Langmuir probe is characterized by a linear plot on semi-log paper. It is seen in Figure 2 that for all values of σ the plots are very close to linear for $\Delta\, i/i_{p1} \leqslant 0.1$. Now electron saturation occurs at $\Delta\, i/i_{p1} = (j_e/j_p) / \sigma$. Therefore the condition that the asymmetrical bipolar probe may be analysed as a single probe is

$$\sigma \geqslant 10\,(j_e/j_p) \qquad (23)$$

We may summarize the result of this analysis as follows:

1. $1 < \sigma < j_e/j_p$. In this mode of operation there is no region of electron saturation; hence, the electron random current density is not measured. In addition the electron temperature that is deduced is representative only of electrons with energies greater than a certain value. There is very little to recommend this mode of operation.

2. $j_e/j_p < \sigma < 10(j_e/j_p)$. Here both electron and positive ion random current densities are measured. The electron temperature is obtained for the complete spectrum of electron energy and it is possible to test for a Maxwellian distribution. Finally, the electron concentration is obtained. This is the preferred mode of operation.

3. $10(j_e/j_p) < \sigma$. The merits of the previous mode are present with the advantage that the data evaluation is somewhat simplified. This is more than outweighed by the disadvantage that, except for very large rockets, the area of the smaller electrode is now very small (less than 10 cm^2 for the Asp) resulting in small saturation currents and large edge corrections.

We conclude that for maximum sensitivity of a rocket-borne Langmuir probe designed to measure electron concentration and temperature the electrode area ratio σ should be chosen to be slightly greater than the expected value of j_e/j_p which is about 200 in the ionosphere.

In the instrumented rockets which are to be fired in the near future the area ratios are:

$$
\begin{aligned}
\text{Nose electrode} \qquad \sigma &= 107 \\
\text{Side electrode} \qquad \sigma &= 1035
\end{aligned}
$$

and the sweep voltage will result in a range of η from about -10 to +10 which is more than adequate to observe the complete current-voltage characteristics.

Table I. Selected Quantities Relevant to the Langmuir Probe

	Ionosphere				Inter-planetary space
Height, h km	150	250	350	700	13
Electron Concentration, n_e cm^{-3} (noon)	2×10^5	10^6	2×10^6	2×10^2	13
Temperature, T°K	1053	1415	1645	1812	200,000
Molecular Weight, M	26.5	25.9	20.6	17.0	1.0
Ratio v_e/v_p (= j_e/j_p)	230	209	196	177	45
Electron Mean Velocity, v_e cm sec^{-1}	2.0×10^7	2.3×10^7	2.4×10^7	2.7×10^7	3.0×10^8
Ion Mean Velocity, v_p cm sec^{-1}	3.8×10^4	11.2×10^4	12.2×10^4	15.0×10^4	6.5×10^6
Electron Random Current Density, j_e amp cm^{-2}	1.5×10^{-7}	9.2×10^{-7}	19.3×10^{-7}	2.2×10^{-7}	1.2×10^{-9}
Ion Random Current Density, j_p amp cm^{-3}	0.7×10^{-9}	4.4×10^{-9}	9.9×10^{-9}	1.2×10^{-9}	2.0×10^{-11}
Floating Potential, V_f volt	-0.48	-0.55	-0.65	-0.81	-.65
Debye shielding length, h cm	0.50	0.26	0.19	0.66	310

* Values up to 700 km from 1959 ARDC Model Atmosphere

Figures

Figure 1. Current-voltage characteristics for asymmetrical bipolar probes. Area ratio σ = 1, 10, 100 and 1000. Probe current ratio i/i_{pl} is ratio of probe current to total positive ion current to larger electrode. Voltage ratio η is ratio of probe voltage to mean electron energy in voltage units. Short horizontal lines indicate electron saturation at smaller electrode for several values of the ratio j_e/j_p which is ratio of electron to positive ion random current density.

Figure 2. Same data as previous figure re-plotted on semi-log coordinates and the ordinate is now $\Delta i/i_{pl}$ = $(i/i_{pl} + 1/\sigma)$. On this method of plotting a single probe analysis leads to a straight line (whose slope is a measure of electron temperature). It is seen that over a wide range of σ that the approximation of single probe analysis is valid for $\Delta i/i_{pl} \leqslant 0.1$.
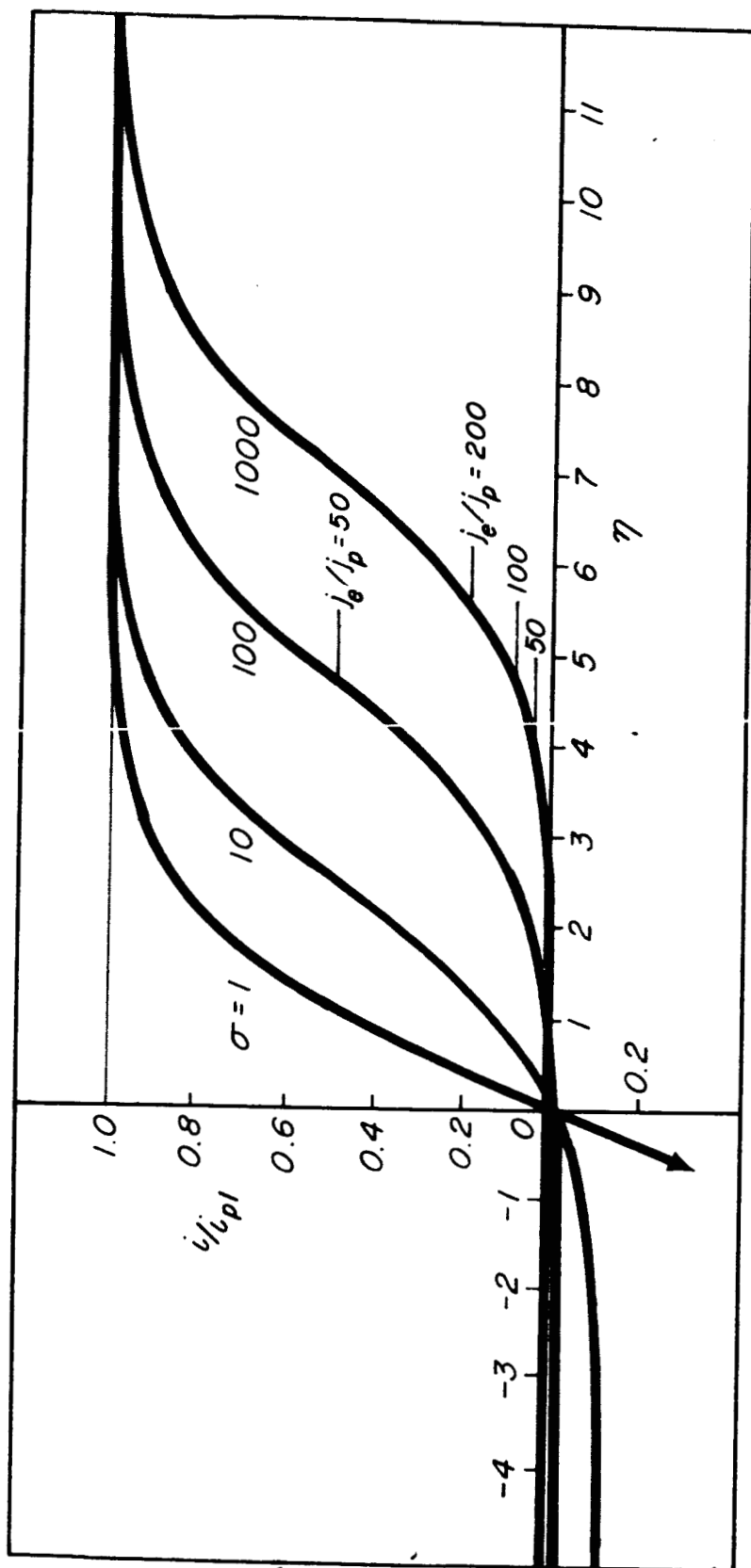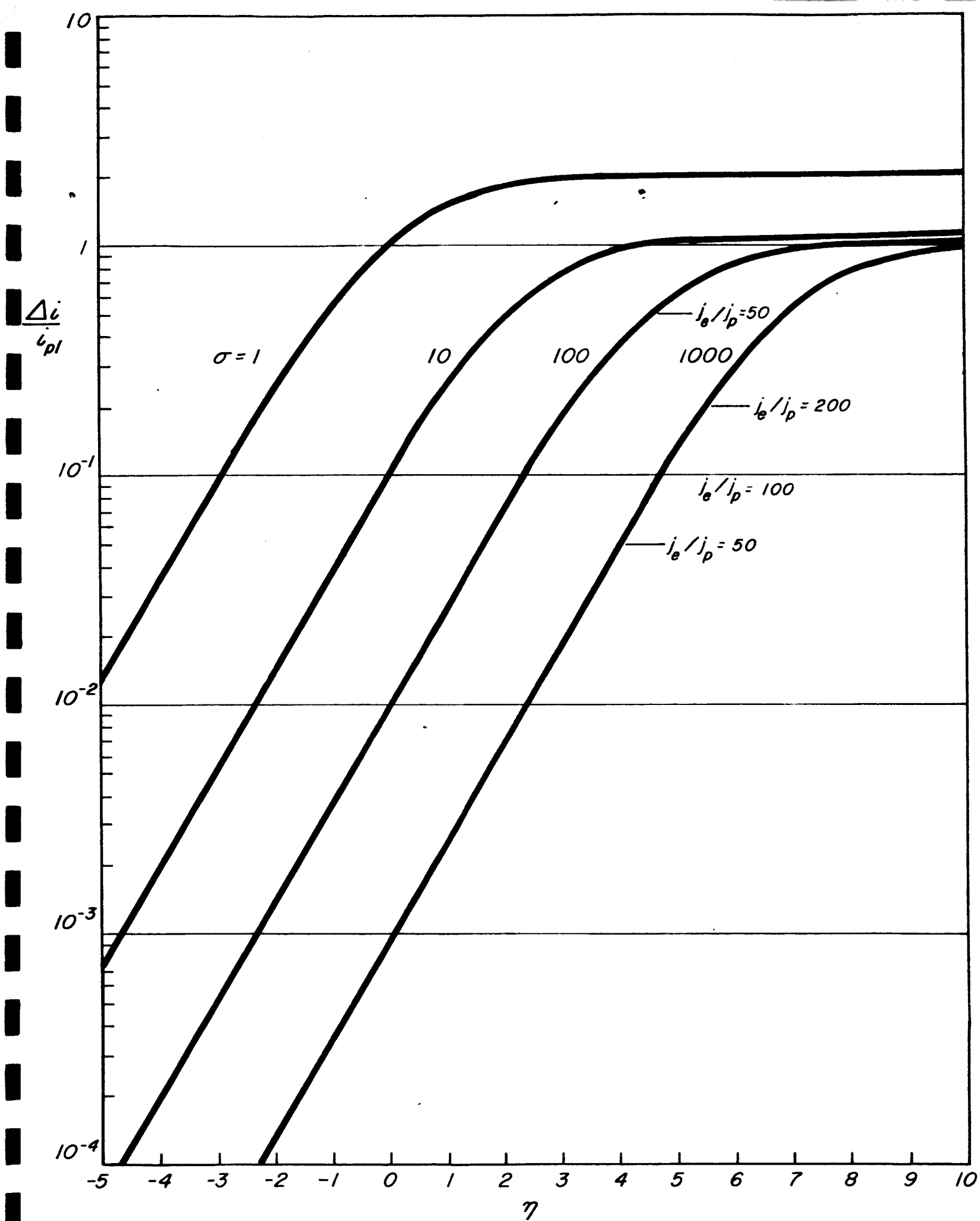
FIGURE 1

FIGURE 2